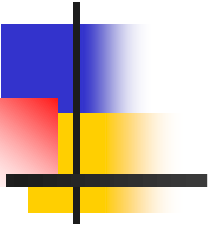


IEng: 5361-Industrial Management and Engineering Economy



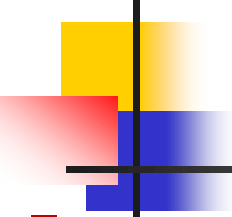
Chapter 4

Inventory Management



Learning Objectives

- *To define inventory*
- *To discuss types and functions of inventory*
- *To discuss inventory classification and counting*
- *To clarify inventory costs*
- *To discuss inventory models*

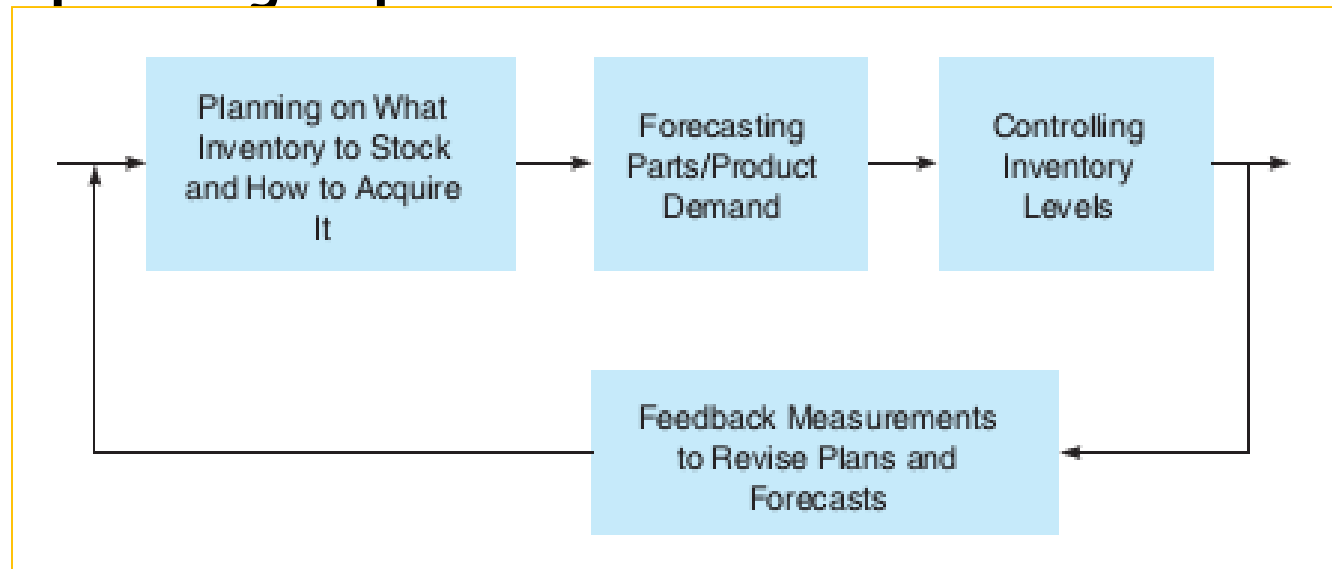


What is inventory?

- **Inventory:** An inventory is an **idle stock of material in store** used to facilitate production or to satisfy customer needs.
- Inventory is any stored resource that is used to satisfy a current or future need.
- Inventory generally refers to the materials in stock. It is also called the **idle resource** of an enterprise.
- **Inventory Management:** Scientific method of finding out **how much stock should be maintained** in order to meet the production demands and be able to provide **right** type of **material** at right **time**, in right **quantities** and at **competitive prices**.

What is inventory?

- **Inventory control** is concerned with achieving an **optimum balance** between two competing objectives.
 - Minimizing the investment in inventory (**Inventory cost**).
 - Maximizing the **service levels** to customer's and it's operating departments.



**Inventory
planning and
control**

What types of inventory?

- **Raw material**

- Purchased but not processed

- **Work-in-process**

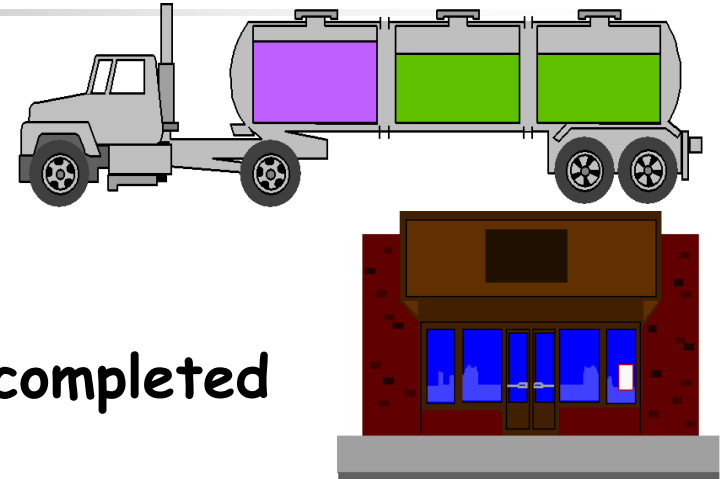
- Undergone some change but not completed

- **Finished goods**

- Completed product awaiting shipment
- Goods-in-transit to warehouses or customers

- **Distribution inventories;**

- finished goods that are located in the distribution system.



What types of inventory?

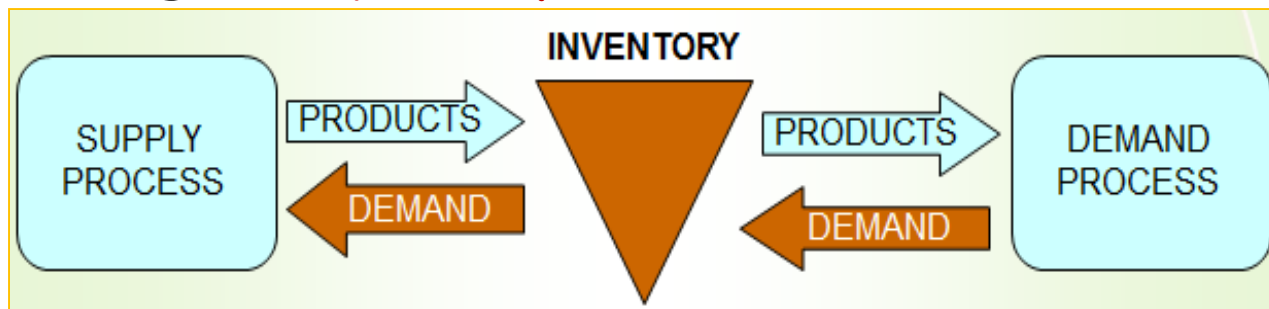
- **Maintenance/repair/operating (MRO)**

- Replacement parts, tools, & supplies
- Necessary to keep machinery and processes productive



What function of inventory ?

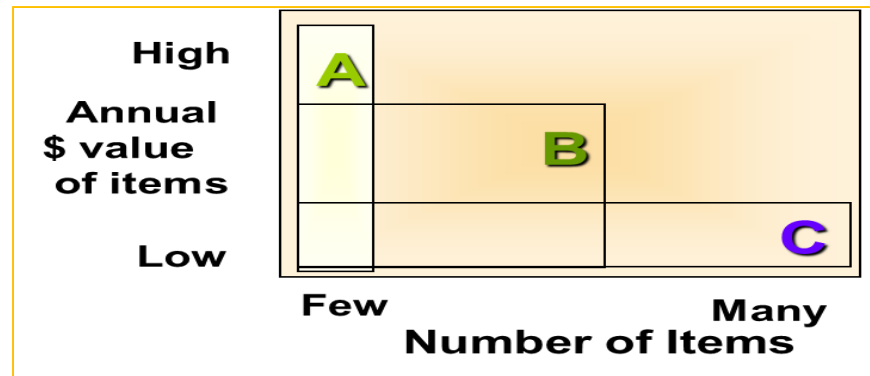
- To meet **anticipated** demand.
- To meet **variation** in product demand.
- To **allow flexibility** in production scheduling.
- To provide a **safeguard** for variation in raw material **delivery time**.
- To protect against **stock-outs**.
- To take advantage of **order cycles** or to take advantage of **quantity discounts**.



What inventory classification and counting?

- **Inventory classification:** is defined as a **process of classifying items** into different categories, thereby directing appropriate attention to the materials in the context of company's viability.
- **ABC classification system:** Classifying inventory according to some **measure of importance** and allocating control efforts accordingly.
- **Importance measure = price * annual sales**

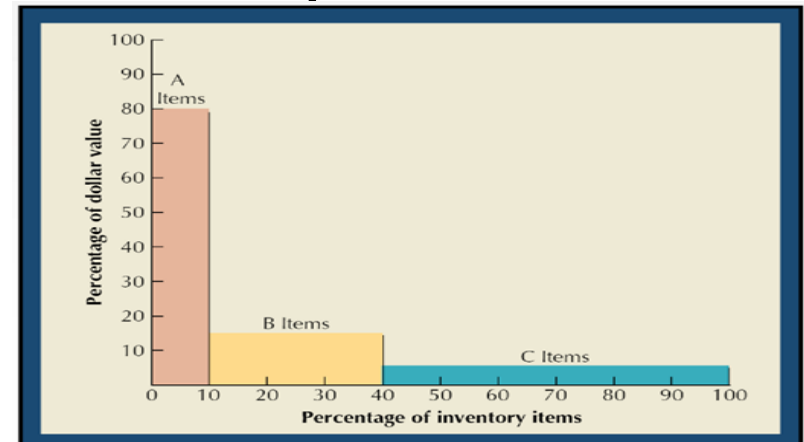
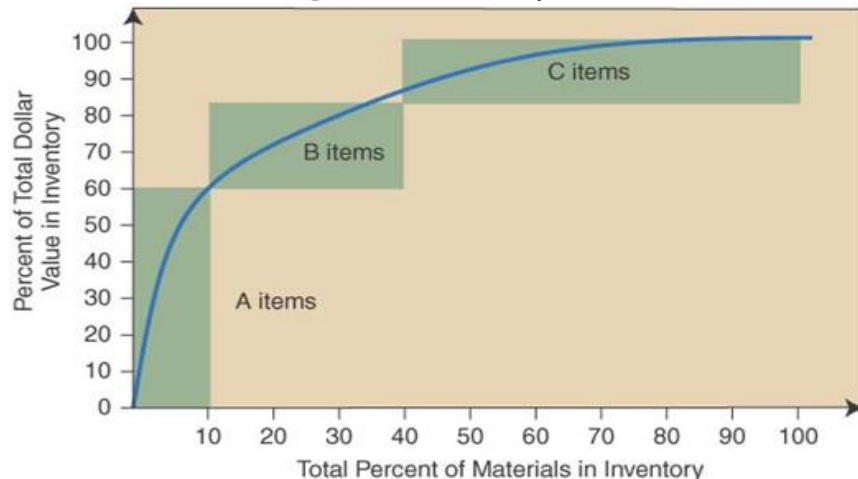
A - very important
B - moderate important
C - least important

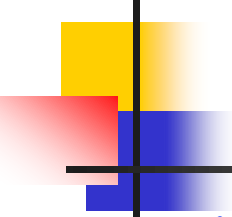


What inventory classification and counting?

ABC Classification Example

- **A Items** - typically 10 - 20% of the items accounting for 70 - 80% of the inventory (Vital few) .
- **B Items** - typically an additional 30% of the items accounting for 15% of the inventory (moderate).
- **C Items** - Typically the remaining 50% - 60% of the items accounting for only 5% - 15% of the inventory value (Trivial many).





What inventory classification and counting?

Steps in ABC Analysis

- a. Compute **Annual Usage** in Dollars for each item and Compute **Total Annual Usage**.
- b. Compute the **percentage of Annual Usage** for each item.
- c. **Sort the list of items** by the percentage of Annual Usage in Dollars, from largest to smallest.
- d. Calculate the Cumulative Percentage of Annual Usage in Dollars for the first item, first 2 items, first 3 items, etc. For the last item, the cumulative % should be 100%. Using Cumulative % as a guide, assign the items to A, B, and C categories.

What inventory classification and counting?

ABC Analysis Example

ABC Problem Data

Item	Unit \$ Value	Annual Usage (in units)
101	12.00	80
102	50.00	10
103	15.00	50
104	50.00	40
105	40.00	80
106	75.00	220
107	4.00	250
108	1.50	400
109	2.00	250
110	25.00	500
111	5.00	450
112	7.50	80
113	3.50	250
114	1.00	1200
115	15.00	300

Solution

ABC Annual Usage Values

Item	Unit \$ Value	Annual Usage (in units)	Annual Usage (\$)
101	12.00	80	960
102	50.00	10	500
103	15.00	50	750
104	50.00	40	2000
105	40.00	80	3200
106	75.00	220	16,500
107	4.00	250	1000
108	1.50	400	600
109	2.00	250	500
110	25.00	500	12,500
111	5.00	450	2250
112	7.50	80	600
113	3.50	250	875
114	1.00	1200	1200
115	15.00	300	4500
Total			\$47,935

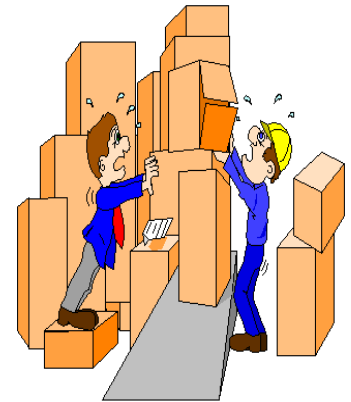
What inventory classification and counting?

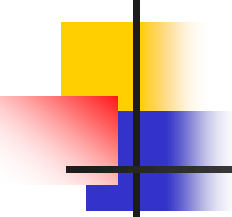
Item	Annual usage			%value	Com.% Value	Categories
	Units \$ Value	in units	value			
106	75	220	16500	34.42161	34.42161	A
110	25	500	12500	26.07698	60.49859	A
115	15	300	4500	9.387713	69.8863	A
105	40	80	3200	6.675707	76.56201	B
111	5	450	2250	4.693856	81.25587	B
104	50	40	2000	4.172317	85.42818	B
114	1	1200	1200	2.50339	87.93157	B
107	4	250	1000	2.086158	90.01773	C
101	12	80	960	2.002712	92.02044	C
113	3.5	250	875	1.825389	93.84583	C
103	15	50	750	1.564619	95.41045	C
108	1.5	400	600	1.251695	96.66215	C
112	7.5	80	600	1.251695	97.91384	C
102	50	10	500	1.043079	98.95692	C
109	2	250	500	1.043079	100	C
15		4160	47935	100		

What inventory classification and counting?

A physical count of items in inventory

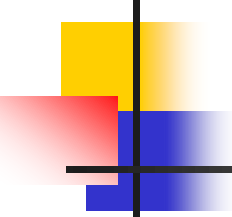
- **Periodic/Cycle Counting System:** Physical count of items made at periodic intervals.
 - How much accuracy is needed?
 - When should cycle counting be performed?
 - Who should do it?
- **Continuous Counting System:** System that keeps track of removals from inventory continuously, thus monitoring current levels of each item.
 - **Two-Bin System** - Two containers of inventory; reorder when the first is empty
 - **Universal Bar Code** - Bar code printed on a label that has information about the item to which it is attached.





What inventory cost?

- **Holding costs:** the costs of holding or “carrying” inventory over time.
 - Housing costs (including rent or depreciation, operating costs, taxes, insurance), Material handling costs (equipment lease or depreciation, power, operating cost), Pilferage, space, and obsolescence, Labor cost.
- **Ordering costs:** the costs of placing an order and receiving goods, Fixed, constant dollar amount incurred for each order placed.



What inventory cost?

- Developing and sending purchase orders, Processing and inspecting incoming inventory, Inventory inquiries, Utilities, phone bills, and so on for the purchasing department, Salaries and wages for purchasing department employees, Supplies such as forms and paper for the purchasing department.
- **Shortage costs:** Loss of customer goodwill, back order handling, and lost sales.
- **Investment costs:** borrowing costs, taxes, and insurance on inventory.



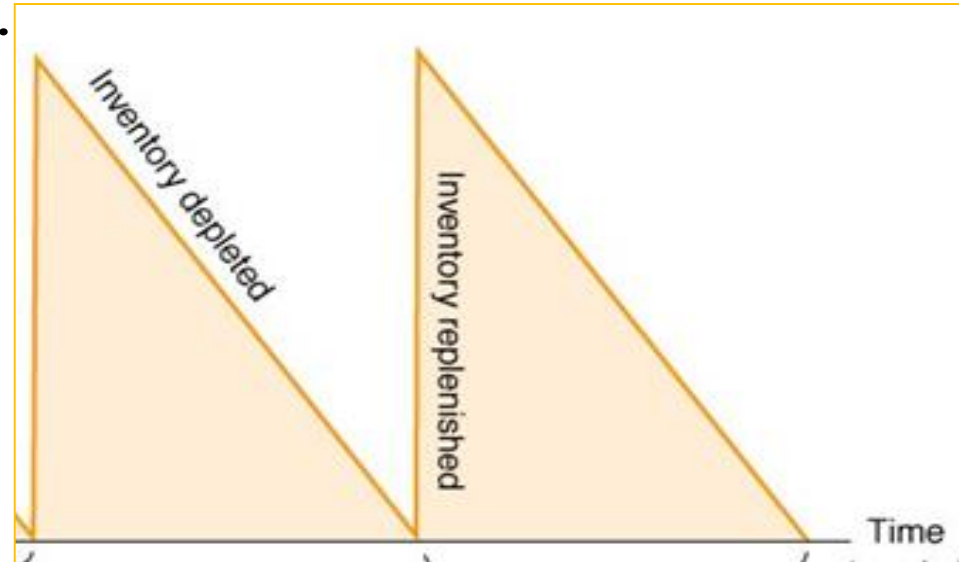
What inventory models?

- Inventory models **deals with** determining **optimum inventory level** that should be kept to keep the **inventory cost to the minimum** and **customer satisfaction** or service level to the **maximum**.
 - When to order?
 - How much to order?
 - How much and when to produce?
 - Level of inventory
- **Inventory Models**
 - Economic Order Quantity (EOQ)
 - Economic Production Quantity (EPQ)
 - Price Discount Models/Price Break Models

What inventory models?

Economic Order Quantity (EOQ)

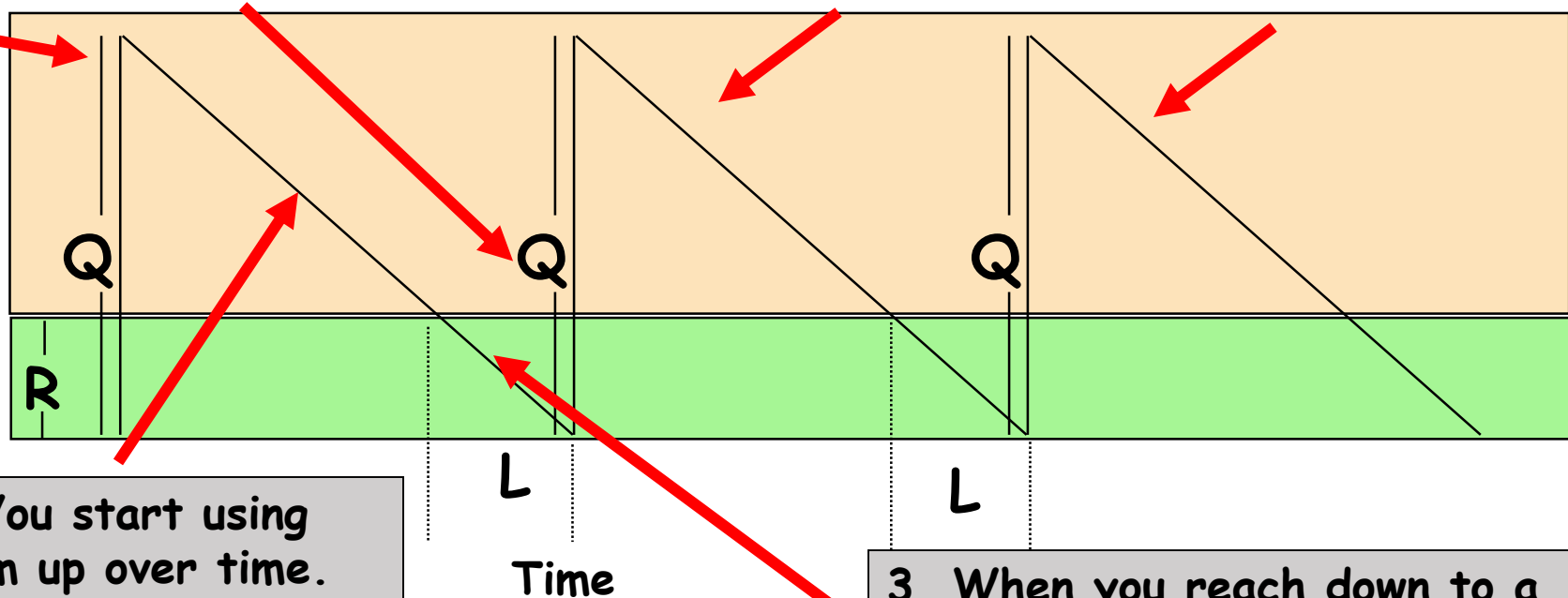
- An **optimizing method** used for determining **order quantity** and **reorder points**.
- Part of **continuous review system** which tracks on-hand inventory each time a withdrawal is made.
- Assumptions:
 - Only one product is involved
 - Annual demand requirement is known and constant.
 - Lead time does not vary.
 - Each order is received in a single delivery.
 - Infinite production capacity



What inventory models?

1. You receive an order quantity Q .

4. The cycle then repeats.



2. You start using them up over time.

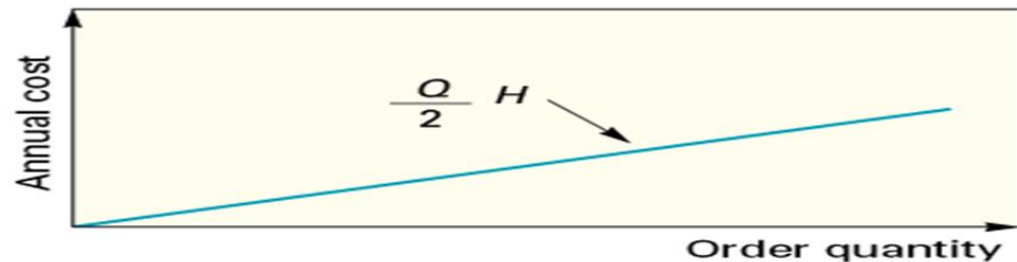
3. When you reach down to a level of inventory of R , you place your next Q sized order.

R = Reorder point
 Q = Economic Order Quantity
 L = Lead time

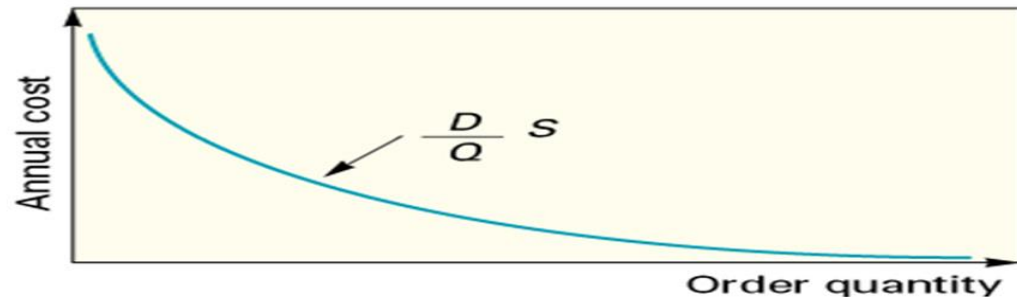
What inventory models?

EOQ Model Costs

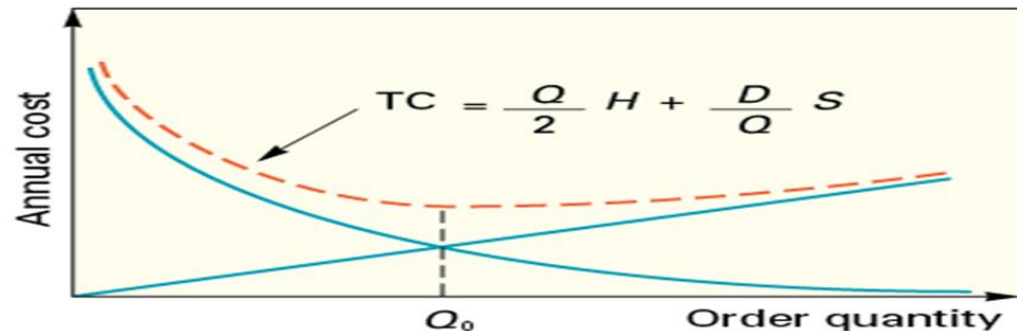
- A. Carrying costs are linearly related to order size.



- B. Ordering costs are inversely and nonlinearly related to order size.



- C. The total-cost curve is U-shaped.



What inventory models?

$$TC_{EOQ} = \left(\frac{D}{Q} S \right) + \left(\frac{Q}{2} H \right) + DC$$

Where

TC = Total annual cost

D = Annual demand

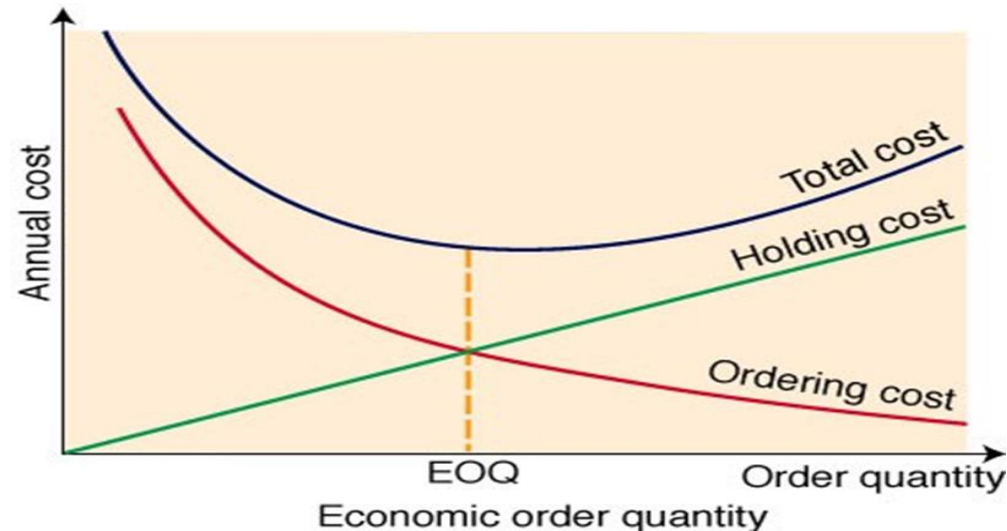
Q = Quantity to be ordered

H = Annual holding cost

S = Ordering or setup cost

C = Cost per unit

Total Annual Costs = Annual Ordering Costs + Annual Holding Costs + Annual purchase Costs



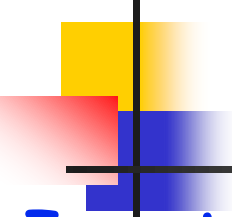
- The **optimal or minimum cost** occurs at the intersection point of **holding cost** and **ordering cost**.
- So using calculus the Q value at this point can be computed.

$$EOQ = \sqrt{\frac{2DS}{H}}$$

Reorder level (R) can be computed as demand during lead time times lead time.

$$R = d * L$$

$$R = d * L + ss \text{ (safety stock)}$$



What inventory models?

Inventory Model Terms

- **Reorder point (R)**: Level of inventory on hand at which the next order should be placed.
- **Cycle Interval (T)**: the total time between one order receipt period and next order receipt.
- **Order frequency (N)**: total number of orders per year (per full inventory cycle).
- **Cycle Inventory ($Q/2$)**: Average inventory kept per cycle interval.

What inventory models?

EOQ example

- Annual Demand = 1,000 units; Days per year considered in average 365; Daily demand = $1000/365$; Cost to place an order = \$10; Holding cost per unit per year = \$2.50; Lead time = 7 days; Cost per unit = \$15.
- With the Given information above, what are the EOQ, reorder point (R), Cycle inventory, Order frequency (N) and Cycle interval (T)?

$$Q_{OPT} = \sqrt{\frac{2DS}{H}} = \sqrt{\frac{2(1,000)(10)}{2.50}} = 89.443 \text{ units or } 90 \text{ units}$$

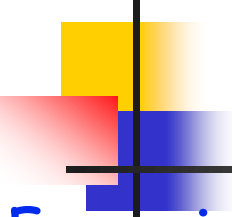
$$d = \frac{1,000 \text{ units/year}}{365 \text{ days/year}} = 2.74 \text{ units/day}$$

$$\text{Cycle Inventory} = Q/2 = 45 \text{ Units}$$

$$R = \bar{d} L = 2.74 \text{ units/day (7 days)} = 19.18 \text{ or } 20 \text{ units}$$

$$N = D/Q = 11.1 \text{ Orders}$$

$$T = Q/d = 32.85 \text{ days}$$



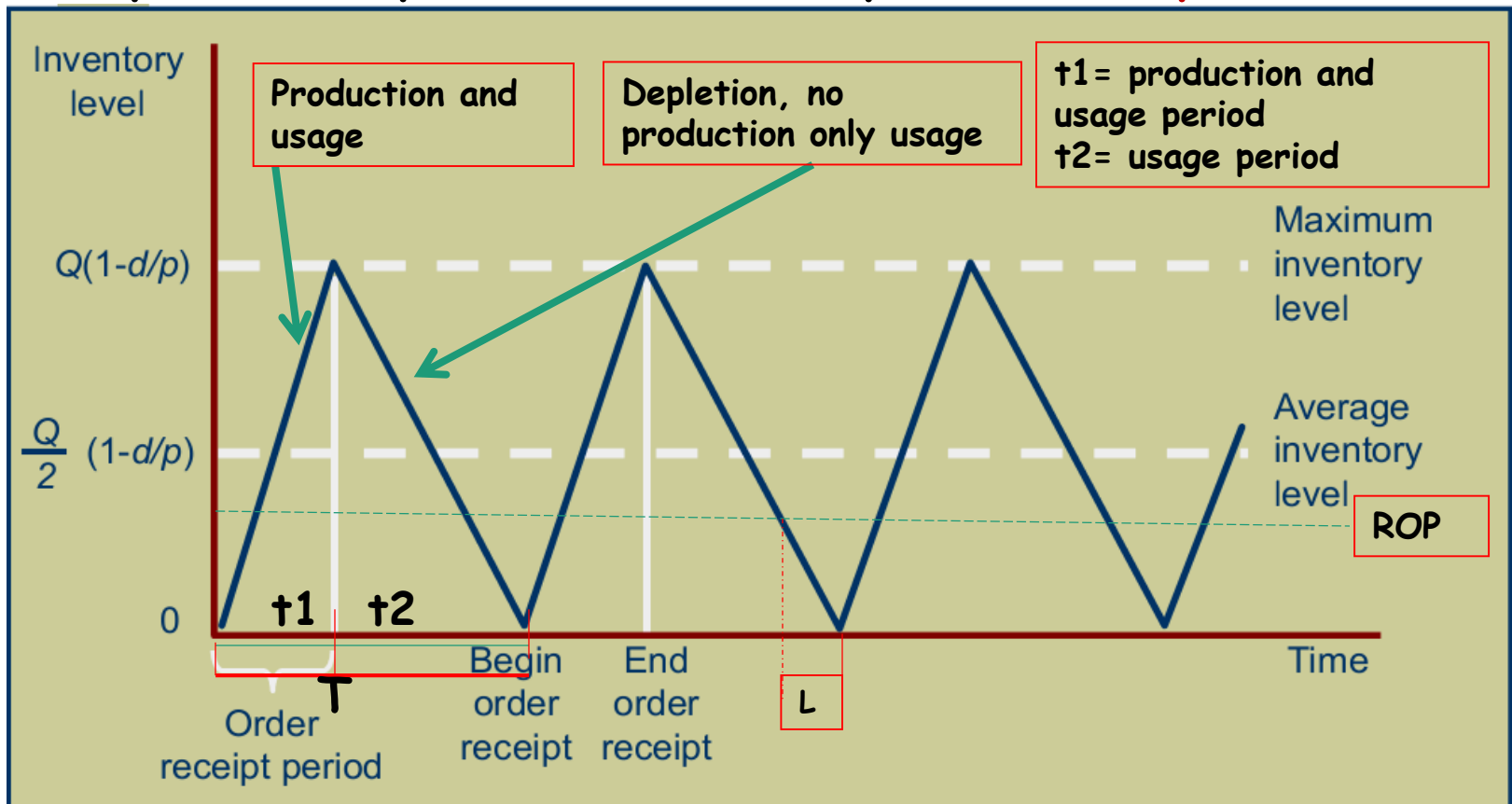
What inventory models?

Economic Production Quantity (EPQ)

- An **optimizing method** used for determining **production quantity** and **reorder points**.
- Production done in **batches** or lots.
- **Capacity to produce** a part **exceeds** the part's **usage or demand rate**.
- Assumptions of EPQ are similar to EOQ **except orders are received incrementally** during production.
 - Only one item is involved
 - Annual demand is known
 - Usage rate **d** is constant
 - Usage occurs continually
 - Production rate **p** is constant
 - Lead time does not vary

What inventory models?

- The Maximum Inventory (I_{max}) is total production during production phase (Q) minus depletion ($Q(d/p)$).



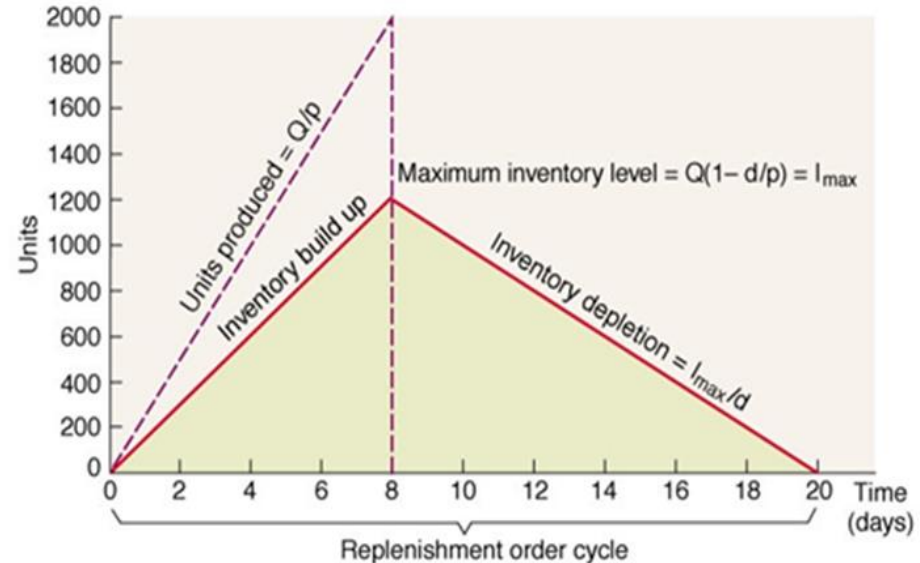
What inventory models?

- The **total cost** at economic production is the sum of **holding** and **setup costs**.
- At the optimal point holding cost and set up **costs are equal**.
- Hence we can derive EPQ formula as follows:

$$TC_{EPQ} = \left(\frac{D}{Q} S \right) + \left(\frac{I_{MAX}}{2} H \right)$$

$$I_{MAX} = Q \left(1 - \frac{d}{p} \right)$$

$$EPQ = \sqrt{\frac{2DS}{H \left(1 - \frac{d}{p} \right)}}$$



The cycle interval (T) is the sum of t1 and t2.

$$t1 = Q/p$$

$$t2 = I_{max}/d$$

By substitution

$$T = Q/d$$

$$R = dL \text{ (Usage rate*Set up time)}$$

$$R = dL + ss \text{ (Safety stock)}$$

What inventory models?

EPQ Example

Annual demand = 18,000 units, **monthly demand=1500 units**,
Production rate = 2500 units/month, Setup cost = \$800, Annual
holding cost = \$18 per unit, **Monthly holding cost=1.5 per unit**,
Setup time= 5 days, No. of operating days per month = 20.

With the given information above, what are the EPQ, I_{max} , reorder point (R), Order frequency (N) and Cycle interval (T), t_1 , t_2 ?

$$EPQ = \sqrt{\frac{2DS}{H \left(1 - \frac{d}{p}\right)}} = 2000 \text{ unit}$$

$$I_{MAX} = Q \left(1 - \frac{d}{p}\right) = 800 \text{ units}$$

$$t_1 = Q/p = 0.8 \text{ month}$$

$$t_2 = I_{max}/d = 0.53 \text{ month}$$

$$T = Q/d = 2000/1500 = 1.33 \text{ months}$$

$$R = d \cdot L = (1500/20) \cdot 5 = 375 \text{ Units}$$

$$N = D/Q = 18000/2000 = 9 \text{ production cycles}$$

$$TC = \text{holding cost} + \text{set up cost} = \$14,400$$

What inventory models?

Price Break (Quantity Discount) Models

- Reduced prices are often available when **larger quantities** are purchased.
 - Trade-off is between reduced product cost and increased holding cost.
 - **Example:** Let the annual demand of item X is 5000 units, Ordering cost is \$49 per order, Holding cost is 20% of unit price per item per year. Determine the **optimal order quantity** that **minimizes the total cost**.
- Here the value of holding cost depends on the amount of units to be ordered, and it varies for each discount.
 - For the above example: $H = i * P$ and Total product cost per year is = PD , where i is percentage of holding cost.

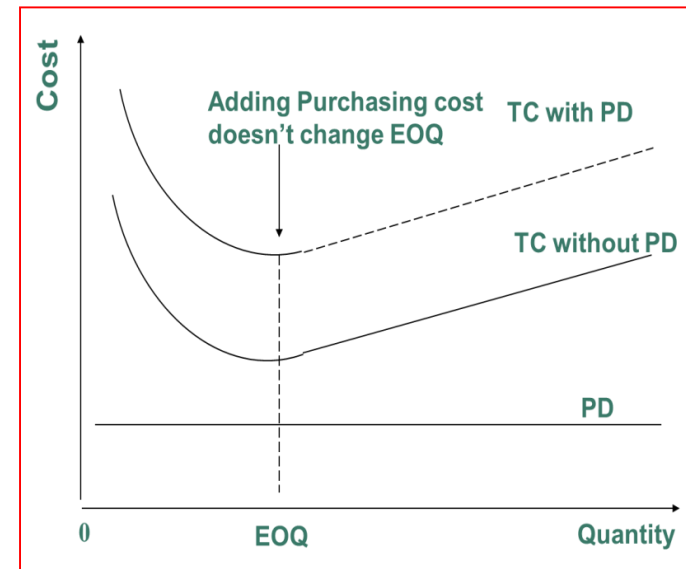
Discount Number	Discount Quantity	Discount (%)	Discount Price (P)*100
1	0 to 999	no discount	\$5.00
2	1,000 to 1,999	4	\$4.80
3	2,000 and over	5	\$4.75

Calculate EOQ* for every discount.

What inventory models?

Price break model procedure

- Calculate the EOQ for each price break.
- Determine whether the EOQ is feasible at that price, check whether the EOQ is in the discount range.
- Will the vendor sell that quantity at that price?
- If yes, stop - if no, continue.
- If EOQ is not feasible select the next higher quantity, minimum on the discount range.
- Calculate the total costs (including total item cost) for the feasible EOQ model of each price break.
- Compare the total cost of each option & choose the lowest TC alternative.





What inventory models?

$$Q^* = \sqrt{\frac{2DS}{iP}}$$

$$Q_1^* = \sqrt{\frac{2(5,000)(49)}{(.2)(5.00)}} = 700 \text{ cars order}$$

$$Q_2^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.80)}} = 714 \text{ items order, adjusted to } 1000 \text{ items.}$$

$$Q_3^* = \sqrt{\frac{2(5,000)(49)}{(.2)(4.75)}} = 718 \text{ items order, adjusted to } 2000 \text{ items.}$$

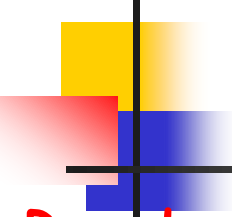
What inventory models?

- The total cost of each price break is computed as follows including the annual product cost and holding cost for each range which depends on product price.

$$TC = QH/2 + DS/Q + PD$$

<i>Discount Number</i>	<i>Unit Price</i>	<i>Order Quantity</i>	<i>Annual Product Cost</i>	<i>Annual Ordering Cost</i>	<i>Annual Holding Cost</i>	<i>Total</i>
1	\$5.00	700	\$25,000	\$350	\$350	\$25,700
2	\$4.80	1,000	\$24,000	\$245	\$480	\$24,725
3	\$4.75	2,000	\$23,750	\$122.50	\$950	\$24,822.50

- Choose the price and quantity that gives the lowest total cost
- Buy **1,000** units at **\$4.80** per unit, Discount number 2.



What inventory models?

Reorder Point and Safety Stock

- **Reorder Point** - When the quantity on hand of an item drops to this amount, the item is reordered. We call it **R**.
- **Determinants of R:**
 - The rate of demand
 - The lead time
 - Demand and/or lead time variability
 - Stock out risk (safety stock)
- **Safety Stock** - Stock that is held in excess of expected demand due to variable demand rate and/or lead time. We call it **SS**.
- **(lead time) Service Level** - Probability that demand will not exceed supply during lead time. We call this cycle service level, **CSL**.
- Risk of a stock out = $1 - (\text{service level})$
- More safety stock means greater service level and smaller risk of stock out.

What inventory models?

■ Without safety stock:

$$R = dL$$

where R = reorder point in units
 d = daily demand in units
 L = lead time in days

■ With safety stocks:

$$R = dL + SS$$

where SS = safety stock in units

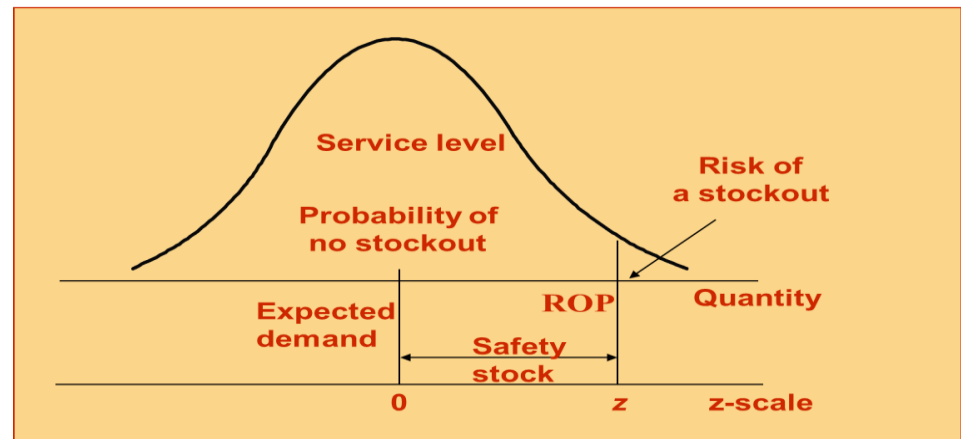
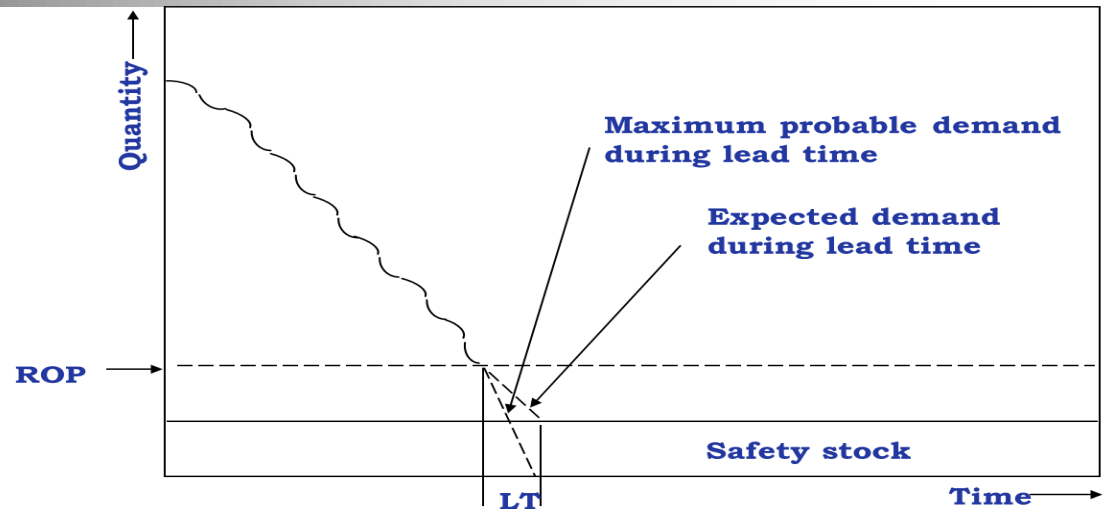
$$SS = z \sigma_{dL}$$

i.e.,

$$R = dL + z \sigma_{dL}$$

z = number of standard deviations associated with desired service level

S = standard deviation of demand during lead time



What inventory models?

This table gives the area under the standardized normal curve from 0 to z , as shown by the shaded portion of the following figure.

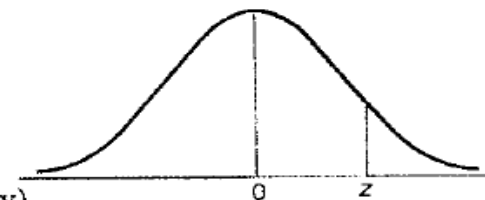
Examples: If z is the standard normal random variable, then

$$\text{Prob}(0 \leq z \leq 1.32) = 0.4066$$

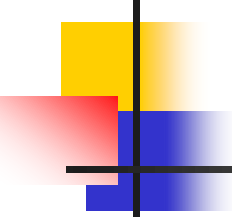
$$\text{Prob}(z \geq 1.32) = 0.5000 - 0.4066 = 0.0934$$

$$\begin{aligned} \text{Prob}(z \leq 1.32) &= \text{Prob}(z \leq 0) + \text{Prob}(0 \leq z \leq 1.32) \\ &= 0.5000 + 0.4066 = 0.9066 \end{aligned}$$

$$\text{Prob}(z \leq -1.32) = \text{Prob}(z \geq 1.32) = 0.0934 \text{ (by symmetry)}$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2518	0.2549
0.7	0.2580	0.2612	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767



What inventory models?

ROP and Safety Stock Example 1

- Daily demand = 20 units, Lead time = 10 days, S.D. of lead time demand = 50 units, Service level (Z) = 90%
- Determine: Safety stock & Reorder point.

$$R = dL + SS$$

$$R = dL + \sigma(1 - SL)$$

The value of **area under normal distribution (Z)** can be found from Z table and for 10% stockout the value of Z is **1.28**.

$$\begin{aligned} R &= 20 \times 10 + 50(\text{norm of } 10\%) \\ &= 200 + 50(1.28) = 264 \\ &\text{units} \end{aligned}$$

$$\begin{aligned} SS &= 50(\text{norm of } 10\%) \\ &= 50(1.28) = 64 \text{ units} \end{aligned}$$

[Z value table.pdf](#)

What inventory models?

ROP and Safety Stock Example 2

- DDLT follows a normal distribution.
- $\mu = 350$, $\sigma = 10$
- They want a 95% service level (i.e. 5% probability of a stock out).
- Here μ is expected demand during lead time with standard deviation of σ .

$$R = dL + SS$$

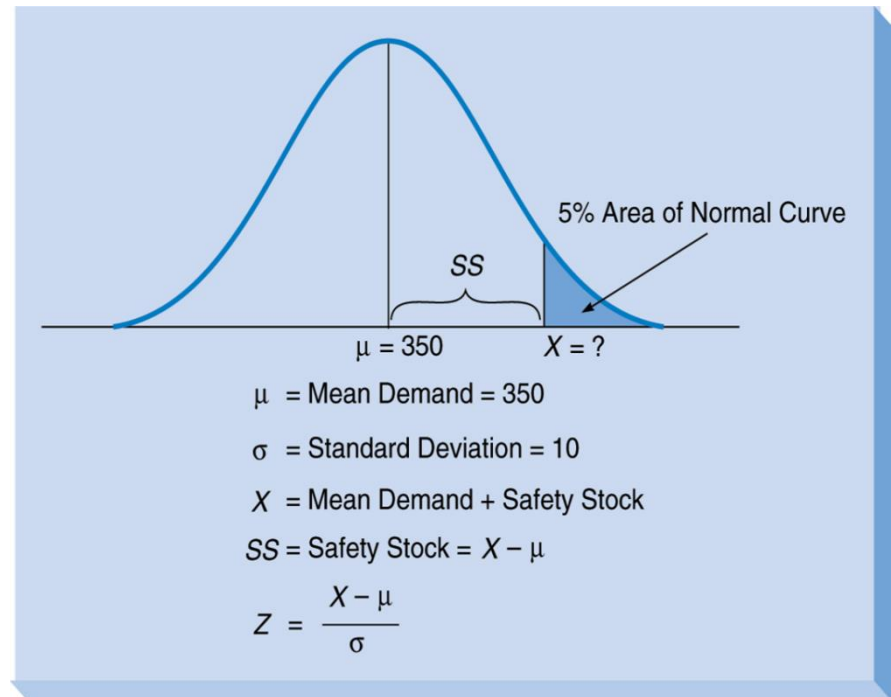
$$R = dL + \sigma(1 - SL)$$

$$\begin{aligned} R &= 350 + 10(\text{norm of } 5\%) \\ &= 350 + 10(1.65) = 366.5 \\ &\text{units} \end{aligned}$$

$$\begin{aligned} SS &= 10(\text{norm of } 5\%) \\ &= 10(1.65) = 16.5 \text{ units} \end{aligned}$$

- Find **R** and **SS**

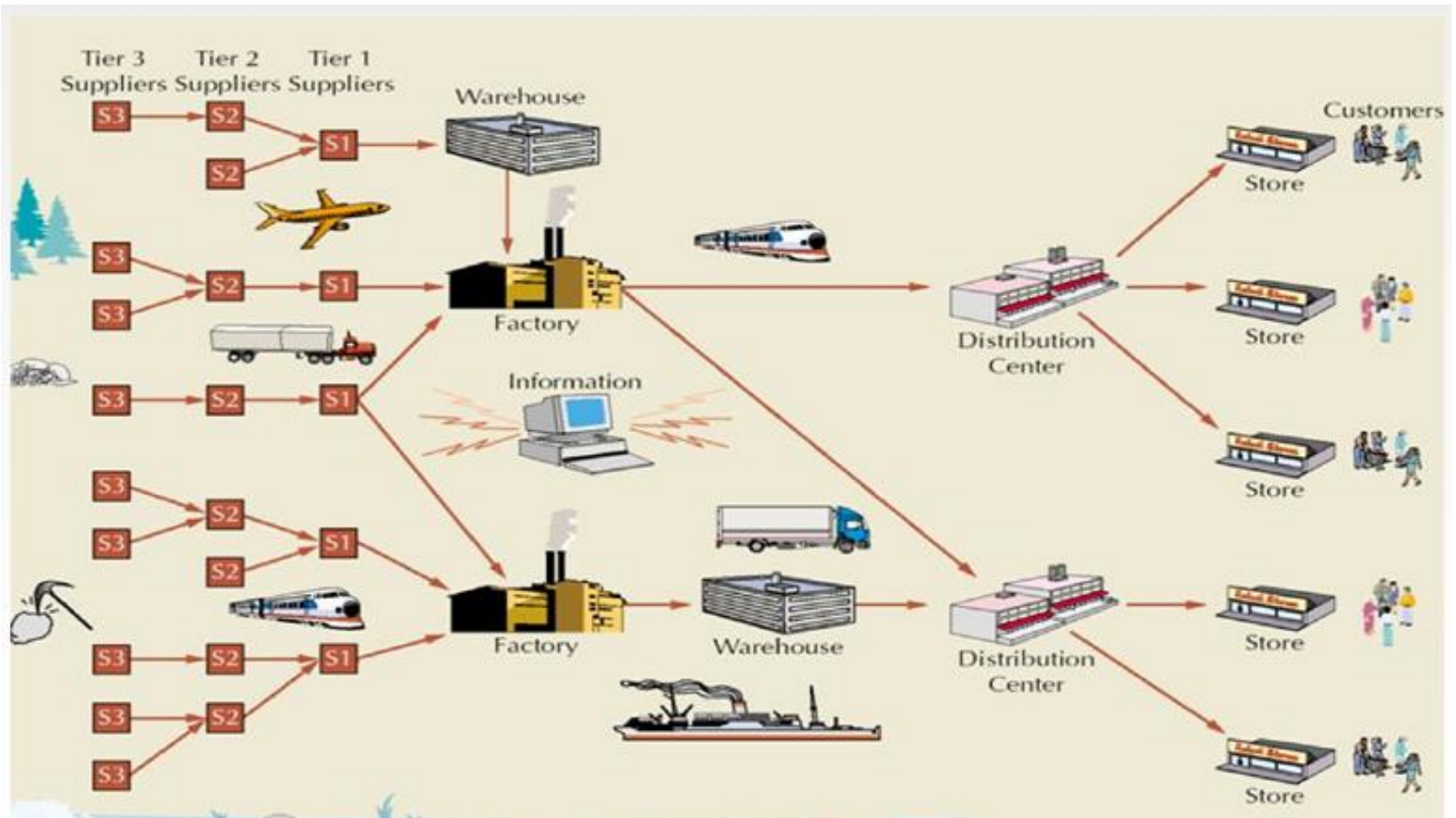
Refer Z Table

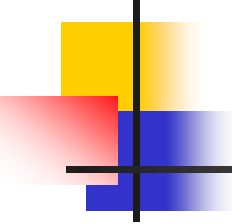


From Z table area under 5% is 1.65

What inventory models?

SCM and Inventory Locations





Thank you

